Seminary 9
SOUND WAVES

SOUND WAVES $y(x,t) = A\cos(kx - \omega t) \quad p(x,t) = BkA\sin(kx - \omega t)$

Summary:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}},$$

Sound waves: Sound consists of longitudinal waves in a medium. A sinusoidal sound wave is characterized by its frequency f and wavelength λ (or angular frequency ω and wave number k) and by its displacement amplitude A. The pressure amplitude p_{max} is directly proportional to the displacement amplitude, the wave number, and the bulk modulus B of the wave medium. (See Examples 16.1 and 16.2.)

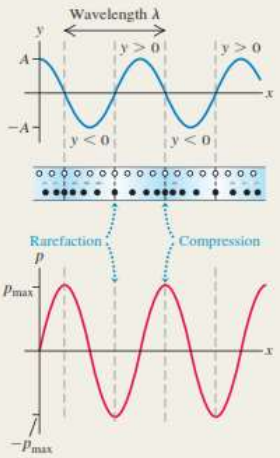
The speed of a sound wave in a fluid depends on the bulk modulus B and density ρ. If the fluid is an ideal gas, the speed can be expressed in terms of the temperature T, molar mass M, and ratio of heat capacities γ of the gas. The speed of longitudinal waves in a solid rod depends on the density and Young's modulus Y. (See Examples 16.3 and 16.4.)

$$p_{max} = BkA \quad (16.5)$$
(sinusoidal sound wave)

$$v = \sqrt{\frac{B}{\rho}} \quad (16.7)$$
(longitudinal wave in a fluid)

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (16.10)$$
(sound wave in an ideal gas)

$$v = \sqrt{\frac{Y}{\rho}} \quad (16.8)$$
(longitudinal wave in a solid rod)



1/ When sound travels from air into water, does the frequency of the wave change? The speed? The wavelength? Explain your reasoning.

2/ Amplitude of a sound wave in air

In a sinusoidal sound wave of moderate loudness, the maximum pressure variations are about 3×10^{-2} Pa above and below atmospheric pressure. Find the corresponding maximum displacement if the frequency is 1000 Hz. In air at normal atmospheric pressure and density, the speed of sound is 344 m/s and the bulk modulus is 1.42×10^5 Pa.

<u>Solution:</u>

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{344 \text{ m/s}} = 18.3 \text{ rad/m}$$

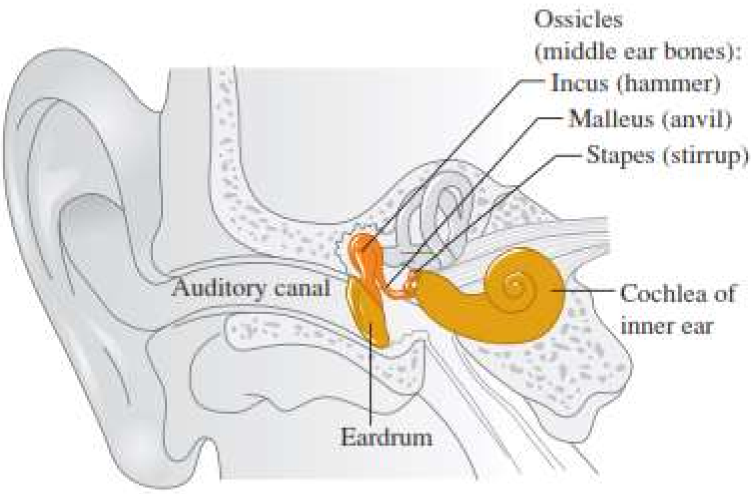
Then from Eq. (16.5), the maximum displacement is

$$A = \frac{p_{max}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(18.3 \text{ rad/m})} = 1.2 \times 10^{-8} \text{ m}$$

This displacement amplitude is only about 1/100 the size of a human cell. The ear actually senses pressure fluctuations; it detects these minuscule displacements only indirectly.

3/ Amplitude of a sound wave in the inner ear

A sound wave that enters the human ear sets the eardrum into oscillation, which in turn causes oscillation of the *ossicles,* a chain of three tiny bones in the middle ear (Fig.). The ossicles transmit this oscillation to the fluid (mostly water with bulk modulus B= 2.18×10^9 Pa) in the inner ear; there the fluid motion disturbs hair cells that send nerve impulses to the brain with information about the sound. The area of the moving part of the eardrum is about $43 mm^2$ and that of the stapes (the smallest of the ossicles) where it connects to the inner ear is about $3.2 mm^2$. For the sound in the previous problem, determine (a) the pressure amplitude and (b) the displacement amplitude of the wave in the fluid of the inner ear, in which the speed of sound is 1500 m/s.



Solution:

(a) *From the area of the eardrum and the pressure amplitude in air found in previous problem, the maximum force exerted by the sound wave in air on the eardrum is* $F_{max} = p_{max\,(air)} S_{eardrum}$.

Then,

$$p_{max\,(inner\ ear)} = \frac{F_{max}}{S_{stapes}} = p_{max\,(air)} \frac{S_{eardrum}}{S_{stapes}}$$

$$= (3.0 \times 10^{-2}\ \text{Pa}) \frac{43\ \text{mm}^2}{3.2\ \text{mm}^2} = 0.40\ \text{Pa}$$

(b) *To find the maximum displacement* $A_{inner\ ear}$, *we use* $A = p_{max}/Bk$

The innder ear fluis is mostly water that has a much greater bulk modulus B than air: B= 2.18×10^9 *Pa.*

The wave in the inner ear has the same angular frequency as the wave in the air because the air, eardrum, ossicles, and inner-ear fluid all oscillate together. But because the wave speed is greater in the inner ear than in the air (1500 m/s versus 344 m/s), the wave vector $k = \omega/v$ *is smaller:*

$$k_{\text{inner ear}} = \frac{\omega}{v_{\text{inner ear}}} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{1500 \text{ m/s}} = 4.2 \text{ rad/m}$$

Putting everything together:

$$A_{\text{inner ear}} = \frac{p_{\max(\text{inner ear})}}{B_{\text{fluid}} k_{\text{inner ear}}} = \frac{0.40 \text{ Pa}}{(2.18 \times 10^9 \text{ Pa})(4.2 \text{ rad/m})}$$
$$= 4.4 \times 10^{-11} \text{ m}$$

Discussion:
In part (a) we see that the ossicles increase the pressure amplitude by a factor of: 13=(43 mm^2)/(3.2mm^2). This amplification helps give the human ear its great sensitivity. The displacement amplitude in the inner ear is even smaller than in the air. But pressure variations within the inner-ear fluid are what set the hair cells into motion, so what matters is that the pressure amplitude is larger in the inner ear than in the air.

4/ The maximum pressure amplitude p_{max} that the human ear can tolerate in loud sounds is about 28 Pa (which is very much less than the normal air pressure of about 10^5 Pa). What is the displacement amplitude *A* for such a sound in air of density ρ= 1.21 kg/m^3, at a frequency of 1000 Hz and a speed of 343 m/s?

Solution:
The displacement amplitude A of a sound wave is related to the pressure amplitude pm of the wave according to p_{max}=BkA.

The sound velocity in the air is $v = \sqrt{\frac{B}{\rho}} \Rightarrow B = \rho v^2$

and $k=2\pi/\lambda=2\pi f/v$ => $A = \frac{p_m}{v\rho(2\pi f)}$

Substituting known data then gives us $A=1.1*10^{-5}m=11\mu m$. *That is only about one-seventh the thickness of a book page. Obviously, the displacement amplitude of even the loudest sound that the ear can tolerate is very small. Temporary exposure to such loud sound produces temporary hearing loss, probably due to a decrease in blood supply to the inner ear. Prolonged exposure produces permanent damage.*

Discussion: The pressure amplitude p_m for the faintest detectable sound at 1000 Hz is $2.8x10^{-5}$ Pa. Proceeding as above leads to A= 1.1x 10^{-11} m or 11 pm, which is about one tenth the radius of a typical atom. The ear is indeed a sensitive detector of sound waves.

5/ The hero of a western movie listens for an oncoming train by putting his ear to the track. Why does this method give an earlier warning of the approach of a train than just listening in the usual way?

6/ You use an electronic signal generator to produce a sinusoidal sound wave in air. You then increase the frequency of the wave from 100 Hz to 400 Hz while keeping the pressure amplitude constant. What effect does this have on the displacement amplitude of the sound wave? (i) It becomes four times greater; (ii) it becomes twice as great; (iii) it is unchanged; (iv) it becomes ½ as great; (v) it becomes ¼ as great.

7/ Diagnostic ultrasound of frequency 4.50 MHz is used to examine tumors in soft tissue. (a) What is the wavelength in air of such a sound wave? (b) If the speed of sound in tissue is 1500 m/s, what is the wavelength of this wave in tissue?

8/ The pressure in a traveling sound wave is given by the equation:

$$\Delta p = (1.50\ \text{Pa})\sin \pi[(0.900\ \text{m}^{-1})\,x - (315\ \text{s}^{-1})t].$$

Find the (a) pressure amplitude, (b) frequency, (c) wavelength, and (d) speed of the wave.

9/ If the form of a sound wave traveling through air is:

$y(x, t) = (6.0\ \text{nm})\cos(kx + (3000\ \text{rad/s})t)$,

how much time does any given air molecule along the path take to move between displacements y=+2.0 nm and y=-2.0 nm?

10/ Find the speed of sound in air at and find the range of wavelengths in air to which the human ear (which can hear frequencies in the range of 20–20,000 Hz) is sensitive. The mean molar T = 20°C, mass for air (a mixture of mostly nitrogen and oxygen) is M =28.8*10^{-3} kg/mol, and the ratio of heat capacities is γ=1.4.

Given: $v = \sqrt{\dfrac{\gamma RT}{M}}$ (speed of sound in an ideal gas) $R = 8.314472(15)\ \text{J/mol}\cdot\text{K}$

SOLUTION

IDENTIFY and SET UP: We use Eq. (16.10) to find the sound speed from γ, T, and M, and we use $v = f\lambda$ to find the wavelengths corresponding to the frequency limits. Note that in Eq. (16.10) temperature T *must* be expressed in kelvins, not Celsius degrees.

EXECUTE: At T = 20°C = 293 K, we find

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.40)(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{28.8\times10^{-3}\ \text{kg/mol}}} = 344\ \text{m/s}$$

Using this value of v in $\lambda = v/f$, we find that at 20°C the frequency f = 20 Hz corresponds to λ = 17 m and f = 20,000 Hz to λ = 1.7 cm.

EVALUATE: Our calculated value of v agrees with the measured sound speed at T = 20°C to within 0.3%.

11/ In a popular and amusing science demonstration, a person inhales helium and then his voice becomes high and squeaky. Why does this happen?

12/ A wire under tension and vibrating in its first overtone produces sound of wavelength .λ What is the new wavelength of the sound (in terms of .λ) if the tension is doubled?